MODELING EXPONENTIAL GROWTH IN SPREADSHEETS

MaTHink 2102

Chris Freiling
and Susan Addington
Cal State San Bernardino

Common Core Standards for Mathematical Practice

- Model with mathematics.
  - Use mathematics to capture some important aspects of a real-world situation
  - Then solve real world problems.
  - How realistic was the model? Adjust if necessary.
- Use appropriate tools strategically.
  - In this presentation, spreadsheets (or calculators if computers not available for students.)

Exponential growth

- The way certain quantities grow (or shrink) over time
  - Animal or plant populations, if there is unlimited food
  - Money, with interest
    - Historical origins in ancient Babylon. If I lend you some cows for a year, next year you will give me the cows back, and also some calves.
  - Repeated zooming in and out in computer maps
  - Repeatedly folding paper
  - Radioactive decay
  - Cooling of a hot liquid

Exponential growth

- Some values of three simple exponential functions

x	y_1	y_2	y_3
0	1	1	1
1	10	2	0.3
2	100	4	0.03
3	1000	8	0.003
4	10,000	16	0.0003

- After linear functions, exponential functions are possibly the most useful in modeling real-world situations.

Interest

- How much do your student know about interest and credit cards?
- "Technology-Rich Mathematics Instruction", Thach and Norman, Teaching Children Mathematics, 2008
  - Used calculators and spreadsheets in a 6th grade class to teach how credit cards work.
  - Started with a worksheet to use with calculators
  - Then moved to a pre-made spreadsheet to enter data, calculations done automatically.

Interest

- Imagine you borrowed $100.
- Lender charges "rent" (interest) for using the money.
- The amount of interest depends on
  - how much you borrowed and
  - how long you kept the money.
- Interest rate is a double ratio: 12% annual interest rate means per cent per year:
  - for every dollar you borrowed and for every year you kept it, you pay 12/100 of a dollar.

Compound interest

- Suppose you borrowed $100 at 12% annual interest. At the end of the year you owe $112.
- If you don't pay the money back, now you will owe interest on $112 at the end of the year.
- At the end of the next year you will owe $112 plus the interest on $112
- …and so on.

Make a table in a spreadsheet

- Type as little as possible; have the spreadsheet do the work
- Use as many columns as you like, to keep your ideas clear. Unlike algebra, you don't want a single expression or equation.

Month	Balance ($)	Interest
0	100	*[have spreadsheet compute the interest]*

Make a table in a spreadsheet

- To tell the spreadsheet to calculate something, start with an = sign.
- Instead of typing the number, type the address of the number.

	A	B	C
1	Month	Balance ($)	Interest
2	0	100	=B2*0.12

	A	B	C
1	Month	Balance ($)	Interest
2	0	100	112

Make a table in a spreadsheet

- Type formulas for A3 and B3

	A	B	C
1	Month	Balance ($)	Interest
2	0	100	12
3	=A2+1	=B2+C2	

- Copy the formula in C2 and paste into C3.

	A	B	C
1	Month	Balance ($)	Interest
2	0	100	12
3	1	112	=B3*0.12

Make a table in a spreadsheet

- When you copy a formula down one row, the address changes down 1 row. Similar for columns.

	A	B	C
1	Month	Balance ($)	Interest
2	0	100	=B2*0.12
3	1	112	=B3*0.12

Make a table in a spreadsheet

- Fill all the formulas down:
  - Highlight cells A3, B3, C3
  - Hover over the lower right corner
  - Drag down as far as you want
  - All rows will be copied with addresses adjusted

	A	B	C
1	Month	Balance ($)	Interest
2	0	100	=B2*0.12
3	=A2+1	=B2+C2	=B3*0.12
4	=A2+1	=B3+C3	=B3*0.12
5	=A2+1	=B4+C4	=B3*0.12

Interpreting the results

- If you don't pay the money back for 10 years, how much will you owe?
- How many years will it take to owe $200? 300?
- Think of some other questions you could answer with this spreadsheet.

Adjust the model

- Actually, interest is often computed monthly, or even daily.
- The monthly interest rate is the annual rate divided by 12.
- Redo your spreadsheet (or make a copy and edit) to show monthly interest calculations.

Adjust the model

- With credit cards, you are required to make a minimum payment: 4% of the balance.
- Adjust your spreadsheet so that you pay 4% after the interest is added (include another column.)
- Start with a bigger amount of money, such as $1000.
- Make more columns to see what would happen if you paid a fixed amount each month, instead of a percentage.

Adjust the model

- This is still not completely realistic:
  - After 4% of your payment is $10, you must pay $10 instead.
  - Interest is actually computed daily.
  - Fees are charged if you are late, and then your interest rate goes up, too.
  - Most people will make more purchases, and not in a pattern.
- How much complication are you willing to build into your spreadsheet?

Make graphs

- See handout or demonstration for how to make a graph ("chart") showing the growth over time.
- Choose scatter plot.

Other situations to model

- Folding paper. A stack of 500 sheets of paper is 2 inches thick. If you repeatedly fold 1 sheet in half, how many layers will there be at each step? How thick will it be?
- Vampires: A lesson done by the Functioning Coefficients group in Project DELTA (7th grade.) Vampires must feed on human blood every day. When a vampire bites a human, the human becomes a vampire. If a vampire enters a town of 10,000 people, how long will it take for everyone to become a vampire? What about the whole state? The whole world? What does this say about whether vampires could exist?

Calculator version of a spreadsheet table

- Use the repeated operation feature of a 4-function calculator.
- Try it out to see what works; calculators vary.
- Press 2x1=
- Now press = again, and again …
- If nothing happened, try 1x2 = = =
- Make a table on paper, fill in with values from calculator.

Videos of exponential growth on the web

- Search YouTube for
  - Exponential growth
  - Folding toilet paper
  - Making croissants
  - Chain reaction
- A short YouTube playlist:
- http://www.youtube.com/playlist?list=PL9EC9D65812CB646C

Thanks!

- Let us know if you use any of these activities in your classroom.
- Chris Freiling: cfreilin@csusb.edu
- Susan Addington: saddingt@csusb.edu